A Data-Driven Fuel Consumption Estimation Model for Airspace Redesign Analysis

1st Ning Hong
Department of Systems Engineering and Engineering Management
City University of Hong Kong
Kowloon, Hong Kong
neil.hong@my.cityu.edu.hk

2nd Lishuai Li
Department of Systems Engineering and Engineering Management
City University of Hong Kong
Kowloon, Hong Kong
lishuai.li@cityu.edu.hk

***Abstract*—A novel data-driven model for fast assessment of terminal airspace redesigns regarding system-level fuel burn is proposed in this paper. When given a terminal airspace design, the fuel consumption model calculates the fleet-wide fuel burn based on the departure/arrival profiles as specified in the design. Then, different airspace designs can be compared and optimized regarding their impact on fuel burn. The fuel consumption model is developed based on the Multilayer Perceptron Neural Network (MLPNN). The model is trained and evaluated using Digital Flight Data Recorder (FDR) data from real operations. We demonstrate the proposed MLPNN method via a case study of Hong Kong airspace and compare its performance with two other regression methods, the robust linear regression (the least median of squares, LMS) method and the ε-insensitive support vector regression (SVR) method. Cross-validation results indicate that the MLPNN performs better than the other two regression methods, with a prediction accuracy of 96.02% on average. Finally, we use the proposed model to estimate the potential fuel burn savings on three standard arrival procedures in Hong Kong airspace. The results show that the proposed model is an effective tool to support fast evaluation of airspace designs focusing on fuel burn.**

Keywords—fuel consumption, airspace design, Flight Data Recorder, Multilayer Perceptron Neural Networks, data-driven approach, Hong Kong airspace

I. INTRODUCTION

Fuel constitutes a large fraction of airlines' operating costs. The global airline industry's fuel bill is estimated to be $149 billion in 2017, accounting for around 21% of operating expenses [1]. Meanwhile, airlines face challenges arising from the need to comply with various environmental regulations. Seeking ways to reduce fuel consumption and minimize the environmental impact is a major goal of the aviation community. The design of airspace around busy airports is of special concern regarding fuel consumption and polluting emissions due to the proximity to cities. A good design will enable aircraft to fly better-optimized trajectories, resulting in less fuel consumption and emissions.

Current fuel consumption models used in practice are primarily based on the Federal Aviation Administration (FAA) airport and airspace simulation model & aviation environmental design tool [2, 3], the International Civil Aviation Organization (ICAO) aircraft engine emissions databank [4], the JEPPESEN total airspace and airport modeler, or the EUROCONTROL Base of Aircraft Data [5]. A few other techniques have been proposed to improve the accuracy of fuel consumption modeling [6-8], but have typically used non-operational data (e.g., from flight manuals, software or ground tests) to model the fuel burn. These studies have therefore not taken operational factors into consideration [9]. For example, the ICAO aircraft engine emissions databank provides values of the fuel flow rate for different aircraft engines in the landing and take-off (LTO) cycle based on ground tests [4], but these values have been proved to differ from those seen in actual operations [10-12]. There are a few studies which have investigated the problem of fuel consumption modeling by using operational flight data [12-17], and the employment of different types of operational data has been shown to further improve estimates of aircraft fuel burn. Nevertheless, all of the above studies only focus on the fuel performance of individual aircraft. More recently, a few efforts have been made to assess the system-level impact of changes in operational procedures on fuel burn and emissions. Jensen, et al. [18] described the development of a method for rapid fleet-wide environmental assessment regarding a number of operational and technology scenarios. However, it required high-fidelity models to build up a database for future rapid summation and cumulative impact calculations. Lastly, fuel burn during taxi has been modeled to evaluate the benefits of congestion mitigation strategies by a number of studies [19-22].

In summary, few studies have focused on a fast tool to evaluate the system-level impact of regional airspace redesign on fuel burn and emissions. The closest line of research to ours is the models proposed by Chati and Balakrishnan to estimate fuel flow rate from trajectory data based on Gaussian Process Regression [9, 17]. Their target variable is fuel flow rate throughout a flight phase, which is unnecessary for the problem to be addressed in this paper - total fuel consumption estimation for airspace redesign purpose.

In this research, we develop a new data-driven model based on multilayer perceptron neural network (MLPNN) for fast assessment of terminal airspace redesigns regarding system-level fuel burn. When given a terminal airspace redesign, the fuel consumption model calculates the average value of fleet-wide fuel consumption based on the relevant profiles. Finally, different airspace designs can be compared and optimized regarding their impact on fuel burn.

This paper is structured as follows: a description of the proposed model is given in Section II. In Section III, we demonstrate the proposed data-driven model using data from real operations. Digital Flight Data Recorder (FDR) data from an international airline are used to build the fuel consumption model. Two other regression methods, the robust linear regression (the least median of squares, LMS) and the ε-insensitive support vector regression (SVR) method are compared with the MLPNN method on prediction accuracy. Lastly, we performed a case study on the fuel consumption improvement potentials regarding different arrival procedures in Hong Kong airspace. Finally, conclusion and future works are given in Section IV.

II. FUEL CONSUMPTION MODELING

In this paper, we propose a novel data-driven model (Fig. 1) for the assessment of system-level fuel consumption estimation about terminal airspace redesigns. The inputs of the model are the information extracted and generated from airspace design profiles of the terminal area (i.e. flight trajectories). The outputs are the average fuel burn for a particular type of aircraft in the terminal area when flying in these designs. Data for training in this model are Digital Flight Data Recorder (FDR) Data, which include hundreds of flight parameters sampled at typically 1Hz, e.g. altitude, airspeed, thrust, fuel flow, etc. With such rich information in FDR data to train a supervised learning model for fuel burn prediction is a straight-forward problem; however, the key challenge is that only a limited number of flight parameters can be used in our model because we need to consider the availability/feasibility to generate those flight parameters from a particular design of terminal airspace. Normally, a design of terminal airspace includes standard departure/arrival routes; aircraft-specific information, i.e. airspeed, the angle of attack, won't be available.

In order to improve the accuracy of the data-driven model, the predictor/explanatory/input variables for modeling are chosen by considering the underlying physics of aircraft and engine operations [23]. These principles suggest that the fuel flow rate depends on the ambient atmospheric density, the true airspeed, the aircraft mass, the wing reference area, and the time. Considering the challenge described in the previous paragraph, we made a few assumptions to build this model. We assume the density is a function of the altitude and the temperature, and the true airspeed is approximately equal to the calibrated airspeed. And we also suppose constant wing area for a particular aircraft type, the relationship between the flying distance and the calibrated airspeed as a surrogate for time, and the aircraft mass to be a function of the instantaneous mass when aircraft starting descending (entering the terminal airspace), the fuel flow rate, and time. Thus, the total fuel burn in terminal airspace functionally depends on the aircraft altitude, the air temperature, the calibrated airspeed, flying distance and the aircraft mass when descending. A key benefit of these simplifying assumptions is that with the exception of the aircraft mass when descending, all of these predictor variables are derivable from aircraft trajectory data (i.e. Automatic Dependent Surveillance-Broadcast, ADS-B data) alone, which are easy to generate based on terminal area airspace design [9].

As mention above, the aircraft altitude, the air temperature, the calibrated airspeed, flying distance and the aircraft mass when descending, are selected as the input variables for estimating the total fuel consumption in terminal airspace. And then, the summary statistics method is applied to process the input parameters. For the air temperature and the calibrated airspeed, the average value within all the time in the terminal airspace is extracted. We choose the difference value between the beginning (entering the airspace) and the end (landing at the airport) altitude values for the altitude, and the instantaneous value when aircraft starting descending for the aircraft mass (Table I).

In the study, Multilayer Perceptron Neural Networks (MLPNNs) is used to build the fuel consumption estimation model. MLPNN are the most commonly used feedforward neural networks due to their fast operation, ease of implementation, and smaller training set requirements [24-26]. The MLPNN consists of three sequential layers: input, hidden and output layers. The training process in MLPNN is carried out in two steps: 1) the inputs are propagated forward through the hidden layers to result in the output values, and then the output values are compared to pre-values in order to estimate the difference, 2) the connection weights were adjusted to optimize the best results with the least difference [27]. Hidden

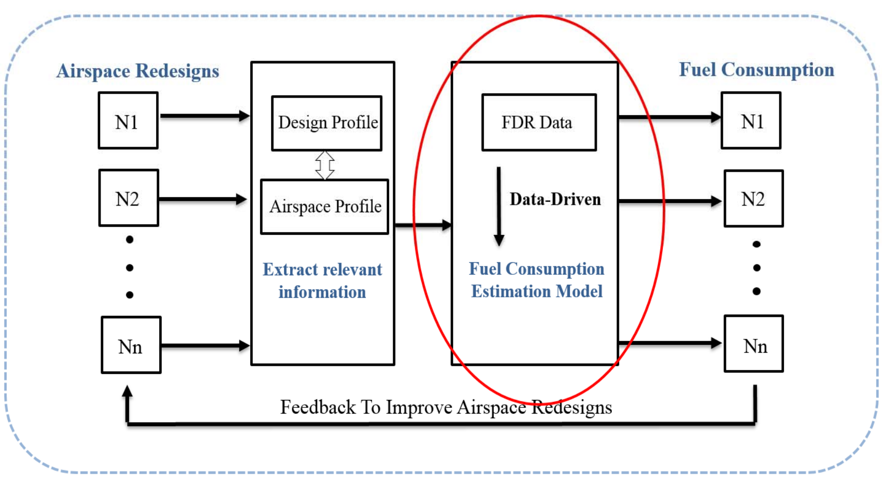


Fig. 1. Fuel consumption estimation model for terminal airspace redesigns

TABLE I. INPUT VARIABLES PROCESSING FOR MODELING

Altitude (x_1)	$x_1 = H_{entering} - H_{landing}$
Air Temperature (x_2)	$x_2 = \frac{1}{n}\sum_{i=1}^{n} T_i$
Calibrated Airspeed (x_3)	$x_3 = \frac{1}{n}\sum_{i=1}^{n} V_i$
Flying Distance (x_4)	$x_4 = \sum_{i=1}^{n} D_i$
Aircraft Mass (x_5)	$x_5 = M_{TOD}$

Note: n represents the number of sample points collected in terminal airspace, H is the altitude value aircraft flying, T is the air temperature, V is the calibrated airspeed, D is the spatial distance between each two adjacent sample points, and M is the aircraft mass.

layers are used for computations and any continuous function can be represented by a neural network that has only one hidden layer [28, 29]. A MLPNN model with insufficient or excessive numbers of neuron in the hidden layer most likely causes the problems of bad generalization and overfitting. Usually, the rule-of-thumb method, which illustrates that the number of hidden layer neurons are 2/3 of the size of the input layer, can be used for determining the number of neurons in the hidden nodes [28, 30]. Based on these principles, the MLPNN with only one hidden layer and three hidden neurons is employed for total fuel burn assessment in our research. (Fig. 2).

Each neuron j in the hidden layer sums up its input signals x_i impinging onto it, after multiplying them by their respective connection weights w_{ji} [31]. The output of each neuron is described as follows:

$$y_j = f(b + \sum w_{ji} x_i) \tag{2.1}$$

where, b is a bias term and f is an activation function using the weighted summations of the inputs. The commonly used activation functions include the sigmoid function, the hyperbolic tangent or a Rectified Linear Unit (ReLu) [32]. In this study, we choose the ReLu as the activation function. ReLu function is non-saturation of its gradient, which greatly accelerates the convergence of stochastic gradient descent compared to the sigmoid and the hyperbolic tangent function. Other advantages of ReLu function are sparse activation, efficient computation and scale-invariant [32, 33].

During the training of the neural network, each w_{ji} weight is adjusted to minimize the difference between the desired and actual values of the output neurons. The minimization objective is normally defined as follow [25, 26]:

$$E = \frac{1}{2}\sum_{j}(y_{dj} - y_j)^2 \tag{2.2}$$

where, y_{dj} is the desired value of the output neuron j and y_j is the actual output of that neuron. Many training algorithms can be used to find the best weights iteratively [25, 26, 34, 35]. Among which, the backpropagation algorithm is most widely used one. However, since the basic version of backpropagation has some drawbacks such as slow convergence [26] or not be able to find the global minimum of the error function [25], a number of variations for the backpropagation were proposed [36-38]. Therefore, we choose to use the backpropagation supported by the Levenberg-Marquardt (LM) algorithm in this study to train the MLPNN model.

III. EXPERIMENTAL RESULTS

In this paper, we perform a case study of the Hong Kong airspace to demonstrate the proposed data-driven model. Digital Flight Data Recorder (FDR) data from an international airline are used to build the fuel consumption model. Two other regression methods, the robust linear regression (the least median of squares, LMS) and the ε-insensitive support vector regression (SVR) method are used to do a comparison with the MLPNN method when building the fuel consumption model. Finally, we identify the potential fuel savings by calculating the difference between current and ideal fuel burn for Hong Kong airspace based on model prediction results.

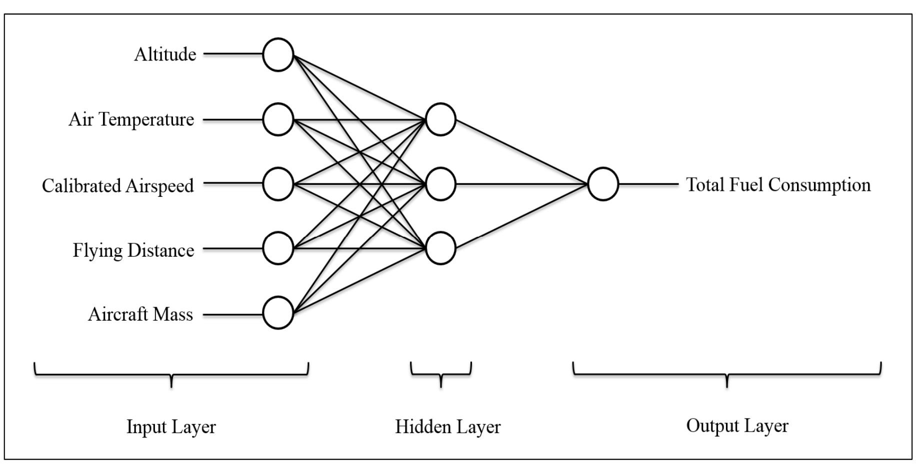


Fig. 2. Fuel consumption estimation model for terminal airspace redesigns

A. Data Description

In this research, the FDR dataset comprises 2686 flights of a specific type of aircraft, Boeing 777-300 ER, from an international airline. All flights land at Hong Kong International Airport during November in 2014, and April - June in 2015.

B. Model Training

Based on the principles discussed in Section II, we select one hidden layer MLPNN with three neurons. And the activation functions used are the ReLu functions. Meanwhile, mean squared error function is set as error function and the backpropagation supported by the Levenberg-Marquardt (LM) algorithm is selected as training algorithm.

Several techniques are used in order to speed up the training of neural networks and reduce over-fitting. We choose to do batch normalization, which normalizing data in the standard manner, with mean zero and standard deviation one. Meanwhile, Glorot's uniform method in [39] is adopted as standard initialization techniques for the weights in MLPNN. Table II reports the parameters setting for the MLPNN.

C. Model Evaluation

In order to assess the MLPNN based method, we compare the proposed method with two other regression methods: 1) the ε-insensitive support vector regression (SVR) method [40-42], a kernel-based technique which is becoming popular in different applicative domains because of its relatively fast training, good performance, and robustness to the overfitting problem [43]; 2) the robust linear regression (the least median of squares, LMS), which is not sensitive to outliers or other violations of the assumption of the usual normal model [44, 45]. The metrics used to evaluate the models are as follows:

- **Mean Absolute Percentage Error (MAPE)**: It's a measure of prediction accuracy of a forecasting method in statistics, and is defined by the formula,

$$M = \frac{100\%}{n}\sum_{t=1}^{n}\left|\frac{A_t - F_t}{A_t}\right| \tag{3.1}$$

  where, A_t is the actual value and F_t is the forecast value.

- **Root Mean Square Error (RMSE):** It's a measure of the differences between values predicted by a model or an estimator and the values observed, and is defined by the formula,

$$R = \sqrt{\frac{\sum_{t=1}^{n}(\hat{y}_t - y_t)^2}{n}} \tag{3.2}$$

  where, y_t is the actual value and $\hat{y}_t$ is the predicted values.

We perform 5-fold leave-one-out cross-validation. Models with low MAPE and low RMSE are preferred. For SVR models, the quadratic kernel is chosen. And, the regularization and kernel width parameters are tuned empirically on the basis of the cross-validation.

TABLE II. PARAMETERS SETTING FOR THE MLPNN

Number of the hidden layer	1
Number of neurons	3
Error function	Mean squared error
Training algorithm	LM algorithm
Activation functions	ReLu
Weight initialization	Glorot's uniform

TABLE III. 5-FOLD CROSS-VALIDATION RESULTS OF THE MLPNN, SVR AND LMS MODELS TRAINED BY FDR DATA IN TERMS OF RMSE AND MAPE

	MLPNN	**SVR**	**LMS**
MAPE	**3.98%**	7.67%	11.38%
Standard Deviation	**0.86%**	0.92%	1.21%
RMSE	**80.12**	86.63	95.53
Standard Deviation	**12.49**	13.29	14.42

D. Model Performance

Table III summarizes the prediction accuracy results of the MLPNN, SVR and LMS models. The results indicate that the MLPNN model performs better than the SVR and the LMS regression methods measured by MAPE and RMSE. Its prediction accuracy is 96.02% on average.

E. Application to Hong Kong Airspace on Potential Fuel Savings

In order to demonstrate the proposed method for an airspace redesign analysis, we use the model to estimate the potential fuel savings of Hong Kong airspace that could be obtained if aircraft can precisely fly the standard arrival procedures.

First, we estimate the ideal fuel burn via the proposed model based on the computer-generated files provided by the airline for three Standard Terminal Arrivals (STARs), namely ABBEY-2B, BETTY-2B, and SIERA-6B/6D, in Hong Kong airspace. A sample of computer-generated files is given in Table IV. As mentioned above, the inputs of fuel consumption estimation model are the aircraft altitude, the air temperature, the calibrated airspeed, flying distance and the aircraft mass when descending. And we could calculate the needed inputs based on computer-generated files except for the value of aircraft mass when descending. We use the average value of 2686 aircrafts initial mass when descending to approximate the aircraft mass in this calculation. The ideal fuel burn of standard arrival procedures is then obtained via the MLPNN model. As shown in Table V, our estimation results are similar to the values generated by the airline's flight planning system, which further validate the accuracy of our model.

Second, we summarize the actual fuel consumption of current operations on these three STARs. In order to identify the flights following these three STARs from the FDR data, we developed a rule-based algorithm to classify the flights based on trajectory information. The classification results are depicted in Fig. 3. The actual fuel burn of flights by each STAR are then obtained from the FDR data directly.

TABLE IV. A SAMPLE OF COMPUTER-GENERATED FILES' FORMAT

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
SIERA	**R473**	**59**	**DSC**	**29/039**	**161**	**163**	**323**	**348**	**15**	**0.02**

Notes:

1. Waypoint name and related longitude, latitude information;
2. Airway name;
3. Minimum route altitude, and 59 represents 5900 feet;
4. Flight level, and DSC represents descent;
5. Wind information, and 29/039 means the wind speed is 39 knots from 290°;
6. True track;
7. Magnetic track;
8. True airspeed, the unit is knots;
9. Ground speed, the unit is knots;
10. Distance to next waypoint, the unit is mile;
11. Time to next waypoint, the unit is minute;

TABLE V. IDEAL FUEL BURN IN TERMINAL AIRSPACE ESTIMATION RESULTS BY THE MLPNN

	ABBEY-2B	BETTY-2B	SIERA-6B/6D
Computer - Ideal Fuel Consumption (kg)	700	900	1200
MLPNN - Ideal Fuel Consumption (kg)	689	887	1180
Estimation Accuracy (%)	98.43	98.56	98.33

TABLE VI. IDEAL FUEL BURN IN TERMINAL AIRSPACE ESTIMATION RESULTS BY THE MLPNN

	ABBEY-2B	BETTY-2B	SIERA-6B/6D
MLPNN - Ideal Fuel Consumption (kg)	689	887	1180
Actual Fuel Consumption (Mean) (kg)	1062.3	1458.7	1657.5
Over Consumption (kg)	373.3	571.7	477.5
Percentage of Over Consumed (%)	54.18	64.45	40.46

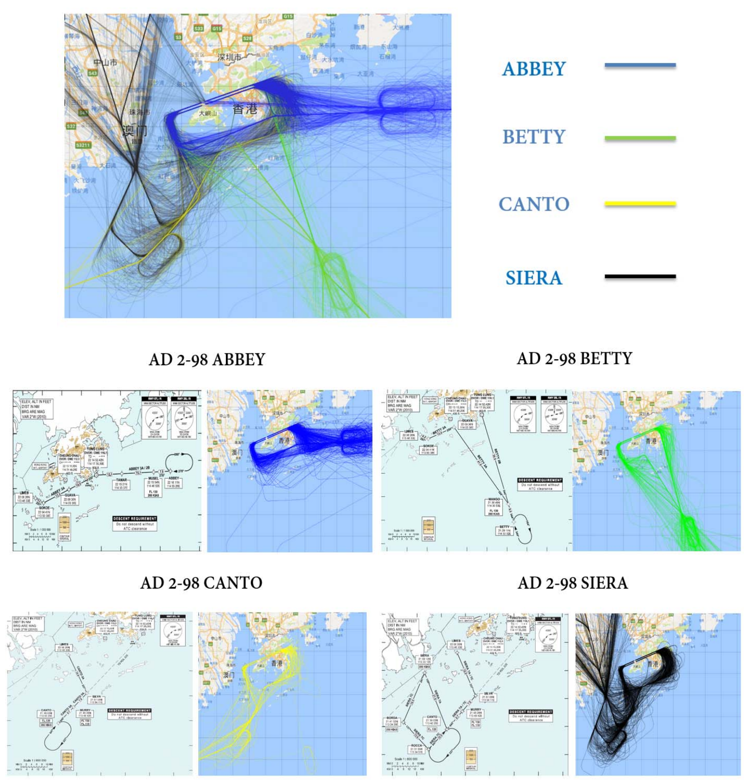
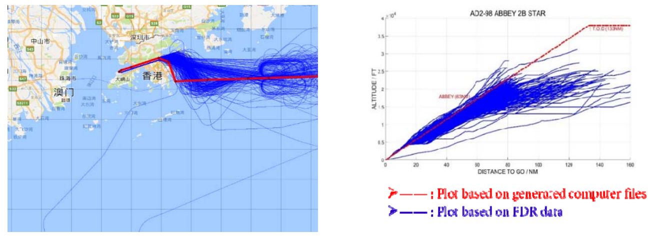
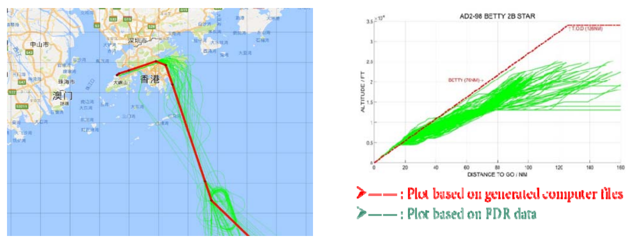


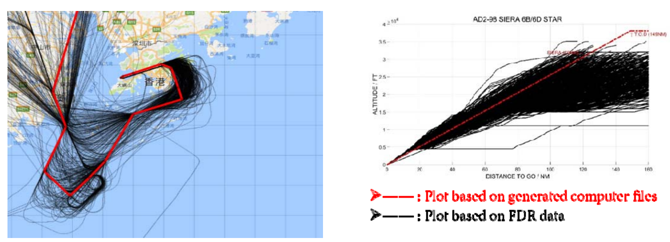
Fig. 3. Four months arrival routes of Boeing 773 ER in Hong Kong Airspace



(a) Horizontal & vertical plot for ABBEY-2B



(b) Horizontal & vertical plot for BETTY-2B



(c) Horizontal & vertical plot for SIERA-6B/6D

Fig. 4. Fuel consumption estimation model for terminal airspace redesigns

Compared with the ideal fuel consumption values, the actual fuel consumption is 54.18%, 64.45% and 40.46% more on average for ABBEY-2B, BETTY-2B, and SIERA-6B/6D, respectively (Table VI). The results indicate that current operations in Hong Kong Airspace are not fuel-efficient. Fig. 4 shows the flight profiles of the actual operations compared with the ideal ones (highlighted in red). Currently, aircraft fly much lower than the planned profiles, which is a major factor that causes the fuel over-consumption of flights that arrive at Hong Kong airport.

IV. Conclusion

In this research, we develop a new data-driven model for fast assessment of terminal airspace redesigns regarding system-level fuel burn. When given a terminal airspace design, the proposed model can calculate the average value of fuel consumption of an aircraft type based on the standard arrival/departure profiles. Using this model, different airspace designs can be compared and optimized regarding their impact on fuel burn.

The proposed model is tested on FDR data from real operations. The performance of this model is better than two other regression methods which based on LMS and SVR, respectively. We also demonstrated the usage of the proposed model in evaluating the potential fuel savings of current Hong Kong airspace design assuming that aircraft can precisely fly the standard arrival procedures.

However, this model is still preliminary. Since the accuracy of fuel consumption estimate is closely linked to that of the aircraft mass. We have not considered how to precisely estimate fuel burn when different types of aircraft flying in the terminal airspace.

Our future work will focus on two parts: 1) improve our fuel burn estimation model to deal with the situation when encountering different types of aircraft, 2) build a trajectory generation model, which could automatically generate trajectories of arrival and/or departure traffic, associated speed profiles and aircraft configurations for a new proposed airspace design.

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