Aircraft Mass Estimation using Quick Access Recorder Data

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Abstract—Aircraft mass is the most important parameter airliners use to determine how much cargo and fuel they should take for the flight to maximize their profit while keep the total weight under the safety limit. For many practical reasons, airliners can not get the accurate aircraft mass data by weighting every cargo and passenger for each flight. Several studies have proposed methods to estimate aircraft mass based on radar data or ADS-B data. But due to the measurement errors in the data, and also due to uncertainties in aircraft configuration, it is hard to calculate accurate aircraft mass data for every flight. In the paper, the quick access recorder data are used for analysis, parameters that are not available in radar data or ADS-B data can now be used to eliminate parameters with large errors in the level flight point-mass dynamics model. Equations are reformulated as a set of overdetermined linear equations with nonlinearly structured errors in system matrix. The set of equations does not depend on inaccurate parameters like thrust and it does not require accurate knowledge of the aircraft like geometry, aerodynamic coefficients, etc. The set of equations is solved by an improved structured nonlinear total least squares method using Monte Carlo method. The method is applied to 120 real flights of Boeing 777-300ER aircraft, the result shows a good accuracy for some type of flights.

Index Terms—aircraft mass estimation, quick access recorder, overdetermined linear equations, structured nonlinear total least squares, Monte Carlo method

I. Introduction

Aircraft mass is the most important parameter for aircraft flight planning, trajectory prediction and performance analysis. Various studies have shown that having inaccurate aircraft mass estimations introduces a significant source of error, which affects all kinds of calculations.

For the research community, data concerning the mass of almost all modern commercial flights are treated as confidential information by airliners, due to its dependence on proprietary information such as load factors and operational strategies. This poses a challenge for the research community. For the airliners, the usefulness of mass data they can get is limited. Because this information is often computed by the flight management system (FMS), rather than measured accurately. For practical reasons, airliners do not weigh cargo and passengers before taking off. Because of this, the mass data provided by FMS may have some level of uncertainty. Accurate aircraft initial mass is required to provide better estimation of fuel consumption and cargo capacity in order to maximize their profit and improve flight safety. A more accurate alternative would be to weigh cargo, fuel, passengers and empty aircraft for each flight, this could be cost ineffective or impractical.

Thus, how to get an accurate estimation of aircraft mass using various available flight data becomes an important question.

On the operational side, [1], [2] tries to calculate the weight of an aircraft based on an approximation of each individual weight component, i.e. aircraft empty weight, fuel weight, and payload weight. However, this method cannot be easily extended to estimate the initial weight of a particular flight, as load factors of individual flights are not publicly known.

On the data science side, different methods have been developed to estimate aircraft mass based on flight data, either from radar data or more recently from ADS-B data. [3] implemented an adaptive estimation method for mass and thrust approximation for the climb phase of aircraft. In a similar approach considering climbing aircraft, [4], [5], developed a least squares method and a machine-learning method, which focused on the climb phase of aircraft. More recently, [6] used ADS-B data from takeoff to estimate the initial mass of an aircraft with two different analytical methods based on least squares. In [7], different mass estimates will first be calculated with appropriate methods for each flight phase, together with fuel flow models. Then, a Bayesian inference approach is established to use these calculations as independent measurements, combining a priori knowledge of initial aircraft mass probability distribution to produce the maximum a posterior estimations. [8] applied statistical machine learning techniques based on gaussian process regression to model the operational takeoff weight by using flight data from the takeoff ground roll. Random disturbances affecting an aircraft's operation (for example, component manufacturing tolerances, turbulence, fluctuations in ambient atmospheric conditions, and component aging and deterioration). In [9], variation in results caused by dependent factors such as prior, thrust and wind are also studied. And the proposed method was validated using 50 test flights of a Cessna Citation II aircraft. The validation results show a mean absolute error of 4.3% of the actual aircraft mass.

For most of these studies, the focus was on a specific phase of the flight (takeoff or climb). It is often not possible to produce a reasonable estimate for individual flights. The possible distribution of aircraft mass is inferred based on a great number of flights. This is not only due to the measurement errors in the data, but also due to uncertainties in aircraft configuration, something that is suggested in all of these studies. To develop a method that can accurately estimate the mass of each flight becomes the focus of this paper.

Another important defect in previous studies is that they all subject to uncertainties in flight data. With more uncertainties in the aircraft states (position, airspeed, vertical rate), the higher noise in the computed mass observations. Such fundamental variance can be improved with better quality data (radar data, wind information, or FMS data). However, this variance will always exist in the estimation.

In particular, since the mass is tightly linked to the thrust in flight dynamics, the uncertainty in thrust will always exists

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in aircraft mass estimations. This can result a big difference between the actual mass and mass computed under maximum thrust profile assumption used by previous methods. Even using the flight dynamics model, with observed aircraft kinematic states and calculated drag, there are multiple possible thrust and mass combinations. That is, both higher and lower thrust-mass combinations may satisfy the equation at the same time (with different levels of error).

For previously used flight data like radar data and ADS-B data. Some important parameters which are crucial to aircraft mass estimations are missing. For example, the flight path angle is used in absence of angle of attack data in flight dynamics, this will introduce certain amount of error. Another important parameter is fuel consumption. When considering an entire flight from takeoff to landing, aircraft mass varies as a function of fuel burn. For an aircraft that is taking off or during initial climb, one can assume the mass is the same as initial mass. However, for mass that is derived from the rest of the climb phase, the descent phase, and the final approach, the consumption of fuel need to be taken into account when inferring the initial mass of the aircraft. For radar data and ADS-B data, the fuel consumption has to be calculated with a fuel burn model, the initial mass of an aircraft can be computed as the sum of flow consumption and the mass computed at a given point of the flight. Although these fuel burn models can be good approximations, there will always be errors.

Some of previous methods requires knowledge of the aircraft like aerodynamic coefficients, thrust profile, etc. Most of these information are not available to researchers, even these information can be get from the aircraft manufacturer or open reference data like BADA3 [10], these information will not be accurate due to different Mach number, Reynolds number, aircraft configurations used in real flight.

In this paper, the quick access recorder (QAR) data are used for analysis, parameters that are not available in radar data or ADS-B data can now be used to eliminate parameters with large errors in the level flight point-mass dynamics model. We reformulate the flight dynamics equations to a set of overdetermined linear equations with uncertainties in both system (data) matrix and the right-hand side (observation) vector. The proposed method doesn't depend on thrust, and it doesn't require knowledge of aircraft specific information like aerodynamic coefficients. The set of equations is solved by an improved structured nonlinear total least squares method using Monte Carlo method. Contrary to most previous studies that use only simulated data (or small samples of real data), we validate the proposed method on a large amount of real data, using months of QAR data from aircraft flying various routes.

The rest of the paper is organized as follows: Section 2 describes the point-mass model used for cruise phase aircraft mass estimation, and formulates the problem as solving a set of overdetermined linear equations with errors in both the nonlinearly structured system matrix and the right-hand vector. Section 3 describes the improved structured nonlinear total least squares method used to find the optimal solutions for the overdetermined linear equations. Section 4 gives the details of the dataset, experimental setup, and results for our experiments. In section 5, conclusions and some perspectives of future research are given.

II. Problem Formulation

For the steady level flight cruise phase, the aircraft dynamics satisfies

$$T\cos\alpha - D - mg\sin\gamma = m\dot{V} \tag{1}$$

$$T\sin\alpha + L - mg\cos\gamma = mV\dot{\gamma} \tag{2}$$

where T, D, L, and W are thrust, drag, lift, and weight forces acting on aircraft, α is angle of attack (AOA).

Using the above two equations to eliminate thrust T

$$W - L - D\tan\alpha = 0 \tag{3}$$

From aerodynamics in [11], lift L and drag D forces can be expressed as

$$L = qSC_L = qS(C_{L_0} + C_{L_\alpha}\alpha) \tag{4}$$

$$D = qS(C_{D_0} + KC_L^2) \tag{5}$$

where, q is dynamic pressure. S is wing reference area. K is drag polar constant. C_L, C_{L_0}, C_{L_α}, C_{D_0} are coefficients of lift, zero-lift, lift slope, zero-lift drag.

Substitute lift and drag in Eq.3

$$W - qS(C_{L_0} + C_{L_\alpha}\alpha) - qS\tan(\alpha)\left(C_{D_0} + K(C_{L_0} + C_{L_\alpha}\alpha)^2\right) = 0 \tag{6}$$

Re-arranging Eq.6

$$W - q(SC_{L_0}) - q\alpha(SC_{L_\alpha}) - q\tan(\alpha)(SC_{D_0} + SKC_{L_0}^2) - q\alpha\tan(\alpha)(2SKC_{L_0}C_{L_\alpha}) - q\alpha^2\tan(\alpha)(SKC_{L_\alpha}^2) = 0 \tag{7}$$

For each time sample i, we have one equation

$$W_i - q_i(SC_{L_0}) - q_i\alpha_i(SC_{L_\alpha}) - q_i\tan(\alpha_i)(SC_{D_0} + SKC_{L_0}^2) - q_i\alpha_i\tan(\alpha_i)(2SKC_{L_0}C_{L_\alpha}) - q_i\alpha_i^2\tan(\alpha_i)(SKC_{L_\alpha}^2) = 0, \quad i = 1, \cdots, m \tag{8}$$

There are m equations and $m + 5$ unknowns $(W_i, S, K, C_{L_0}, C_{L_\alpha}, C_{D_0})$, thus unsolvable.

Consider aircraft weight consists of fuel weight W_f and others W_o, and we can assume W_o to be a constant, that is

$$W_i = W_o + W_{f,i}, \quad i = 1, \cdots, m \tag{9}$$

Then Eq.8 becomes

$$\underbrace{W_o}_{x_1} - q_i\underbrace{(SC_{L_0})}_{x_2} - q_i\alpha_i\underbrace{(SC_{L_\alpha})}_{x_3} - q_i\tan(\alpha_i)\underbrace{(SC_{D_0} + SKC_{L_0}^2)}_{x_4} t - q_i\alpha_i\tan(\alpha_i)\underbrace{(2SKC_{L_0}C_{L_\alpha})}_{x_5} - q_i\alpha_i^2\tan(\alpha_i)\underbrace{(SKC_{L_\alpha}^2)}_{x_6} = -W_{f,i}, \quad i = 1, \cdots, m \tag{10}$$

There are m equations and 6 unknowns $(x_i, i = 1, \cdots, 6)$. Usually $m \gg 6$, so we have a group of overdetermined linear equations. In matrix form

$$\boldsymbol{Ax} = \boldsymbol{b} \tag{11}$$

where $\boldsymbol{A} \in \mathbb{R}^{m\times n}$ and $\boldsymbol{b} \in \mathbb{R}^{m\times 1}$ are the given data and $\boldsymbol{x} \in \mathbb{R}^{n\times 1}$ is unknown.

$$\boldsymbol{A} = \begin{bmatrix} -1 & q_1 & q_1\alpha_1 & q_1\tan\alpha_1 & q_1\alpha_1\tan\alpha_1 & q_1\alpha_1^2\tan\alpha_1 \\ -1 & q_2 & q_2\alpha_2 & q_2\tan\alpha_2 & q_2\alpha_2\tan\alpha_2 & q_2\alpha_2^2\tan\alpha_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -1 & q_m & q_m\alpha_m & q_m\tan\alpha_m & q_m\alpha_m\tan\alpha_m & q_m\alpha_m^2\tan\alpha_m \end{bmatrix} \tag{12}$$

$$\boldsymbol{b} = [W_{f,1} \quad W_{f,2} \quad \ldots \quad W_{f,m}]^T \tag{13}$$

$$\boldsymbol{x} = \big[W_o \quad SC_{L_0} \quad SC_{L_\alpha} \quad SC_{D_0} + SKC_{L_0}^2 \quad 2SKC_{L_0}C_{L_\alpha} \quad SKC_{L_\alpha}^2\big]^T \tag{14}$$

Since $\alpha_i \approx \tan(\alpha_i)$ for small α_i, the third and fourth columns of $\boldsymbol{A}$ are close, makes matrix $\boldsymbol{A}$ poorly conditioned. To better ensure $\boldsymbol{A}$ has full column rank, the third and fourth columns are combined as one.

$$\begin{aligned} &\underbrace{W_o}_{x_1} - q_i \underbrace{(SC_{L_0})}_{x_2} \\ &\quad \underbrace{- q_i\alpha_i \underbrace{(SC_{L_\alpha})}_{\cancel{x_3}} - q_i\tan(\alpha_i) \underbrace{(SC_{D_0} + SKC_{L_0}^2)}_{\cancel{x_4}}}_{x_3} \\ &\quad - q_i\alpha_i\tan(\alpha_i) \underbrace{(2SKC_{L_0}C_{L_\alpha})}_{\cancel{x_5}\, x_4} \\ &\quad - q_i\alpha_i^2\tan(\alpha_i) \underbrace{(SKC_{L_\alpha}^2)}_{\cancel{x_6}\, x_5} = -W_{f,i}, \quad i = 1, \cdots, m \end{aligned} \tag{15}$$

matrix $\boldsymbol{A}$ and vector $\boldsymbol{x}$ becomes

$$\boldsymbol{A} = \begin{bmatrix} -1 & q_1 & q_1\alpha_1 & q_1\alpha_1\tan\alpha_1 & q_1\alpha_1^2\tan\alpha_1 \\ -1 & q_2 & q_2\alpha_2 & q_2\alpha_2\tan\alpha_2 & q_2\alpha_2^2\tan\alpha_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -1 & q_m & q_m\alpha_m & q_m\alpha_m\tan\alpha_m & q_m\alpha_m^2\tan\alpha_m \end{bmatrix} \tag{16}$$

$$\boldsymbol{x} = \big[W_o \quad SC_{L_0} \quad SC_{L_\alpha} + SC_{D_0} + SKC_{L_0}^2 \quad 2SKC_{L_0}C_{L_\alpha} \quad SKC_{L_\alpha}^2\big]^T \tag{17}$$

Since both $\boldsymbol{A}$ and $\boldsymbol{b}$ are measured with possible errors, and typically there is no exact solution for $\boldsymbol{x}$ with $m > n$, so that an approximate solution is sought for.

$$\boldsymbol{Ax} \approx \boldsymbol{b} \tag{18}$$

III. Optimal Solution for Overdetermined Linear Equations with Errors

Overdetermined linear equations in Eq.(18) arise in many practical applications. Typical applications comes from signal processing, like [12]–[15].

If only the right hand side vector $\boldsymbol{b}$ is subjected to error, the system matrix $\boldsymbol{A}$ is known without error. The ordinary least squares (LS) solution $\boldsymbol{x}_{ls}$ is such that

$$\begin{aligned} \min_{\boldsymbol{x},\Delta\boldsymbol{b}} \quad & \|\Delta\boldsymbol{b}\|_2 \\ s.t. \quad & \boldsymbol{Ax} = \boldsymbol{b} + \Delta\boldsymbol{b} \end{aligned} \tag{19}$$

If both $\boldsymbol{A}$ and $\boldsymbol{b}$ are measured with error, the total least squares (TLS) solution $\boldsymbol{x}_{tls}$ looks for the minimal (in the Frobenius norm sense) corrections $\Delta\boldsymbol{A}$ and $\Delta\boldsymbol{b}$ on the given data $\boldsymbol{A}$ and $\boldsymbol{b}$ that make the corrected system of equations $\hat{\boldsymbol{A}}\boldsymbol{x} = \hat{\boldsymbol{b}}$, $\hat{\boldsymbol{A}} := \boldsymbol{A} + \Delta\boldsymbol{A}$, $\hat{\boldsymbol{b}} := \boldsymbol{b} + \Delta\boldsymbol{b}$ solvable.

$$\begin{aligned} \min_{\boldsymbol{x},\Delta\boldsymbol{A},\Delta\boldsymbol{b}} \quad & \|[\Delta\boldsymbol{A} \ \Delta\boldsymbol{b}]\|_F \\ s.t. \quad & (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x} = \boldsymbol{b} + \Delta\boldsymbol{b} \end{aligned} \tag{20}$$

The TLS solution $\boldsymbol{x}_{tls}$ is a consistent estimator of true $\boldsymbol{x}$ if $\boldsymbol{A}$ and $\boldsymbol{b}$ are zero mean random and with a multiple of the identity covariance matrix. But in many of real applications the matrix $\boldsymbol{A}$ has a special structure, the TLS solutions do not preserve the special structure. In fact, using the TLS approach the matrix $\Delta\boldsymbol{A}$ will typically be dense, with no special structure, even when A is Toeplitz or sparse. Thus, even those elements of $\Delta\boldsymbol{A}$ that should remain zero will typically become nonzero. Also in some situations, the use of a norm other than the Frobenius norm may be preferable. For example, if the data contains outliers, that is there may be large random errors in some elements of matrix $\boldsymbol{A}$ and in vector $\boldsymbol{b}$, ℓ_1 or ℓ_∞ norm might be more suitable.

The structured total least norm (STLN) method in [16] takes full advantage of the special structure of a given matrix $\boldsymbol{A}$. Our case is a more difficult structured approximation problem, because the elements of $\boldsymbol{A}$ are nonlinear differentiable functions of a parameter vector $\boldsymbol{\beta}$, this extension is called structured nonlinear total least norm (SNTLN) in [17]. In particular, when $s(\leq m \times n)$ elements of $\boldsymbol{A}$ subject to error, a vector $\boldsymbol{\alpha} \in \mathbb{R}^{s\times 1}$ is used to represent the corresponding elements of the error matrix $\Delta\boldsymbol{A}$. Furthermore, if many elements of $\Delta\boldsymbol{A}$ must have the same value, then s is the number of different such elements. The residual vector $\boldsymbol{r} = \boldsymbol{b} - (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x}$ is now a function of $\boldsymbol{\beta}$ and $\boldsymbol{x}$, so $\boldsymbol{r} = \boldsymbol{r}(\boldsymbol{\beta}, \boldsymbol{x})$. Let $\boldsymbol{\beta_0}$ be the initial estimate of the optimum parameter vector $\boldsymbol{\beta}$, $\boldsymbol{D}$ be a $(s\times s)$ diagonal weighting matrix. Then the SNTLN problem can be stated as follows:

$$\begin{aligned} \min_{\boldsymbol{x},\boldsymbol{\beta}} \quad & \left\| \begin{bmatrix} \boldsymbol{r}(\boldsymbol{\beta}, \boldsymbol{x}) \\ \boldsymbol{D}(\boldsymbol{\beta} - \boldsymbol{\beta_0}) \end{bmatrix} \right\|_p \\ s.t. \quad & (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x} = \boldsymbol{b} + \Delta\boldsymbol{b} \end{aligned} \tag{21}$$

where $\|\cdot\|_p$ is the vector p-norm, for $p = 1, 2, or\ \infty$.

The SNTLN algorithm is effective for small scale problems. For large scale problems with large m, especially when the size of $\boldsymbol{\alpha}$ is comparable to $m \times n$, the algorithm is computationally expensive. For $p = 1\ or\ \infty$, the algorithm has to solve a linear programming problem for each iteration, which is considerably slower than $p = 2$ that has a closed form solution. Although solutions obtained using ℓ_1 or ℓ_∞ norm are very robust with respect to large data errors.

Also because of the small sampling interval, the adjacent rows of system matrix $\boldsymbol{A}$ and vector $\boldsymbol{b}$ are very close, this also makes matrix $\boldsymbol{A}$ or $[\boldsymbol{A}\ \boldsymbol{b}]$ poorly conditioned, result in large error in the solution $\boldsymbol{x}_{sntln}$. Unlike the close columns caused by modeling, which can be fix by use a improved model, this can not be avoided by using original SNTLN algorithm.

To solve two above problems, we propose an improved structured nonlinear total least squares algorithm based on the original SNTLN algorithm in [17] using the Monte Carlo method. For large scale problems with large m, and with large errors (outliers) in some rows of Eq.(18), in stead of solving m rows of $\boldsymbol{Ax} \approx \boldsymbol{b}$ at once, we randomly choose k rows from it each time, solves the partial problem, and calculate the average of all

Monte Carlo experiments. The improved structured nonlinear total least squares algorithm can be summarized as

Algorithm 1 Improved structured nonlinear total least squares

Input: Matrix $\boldsymbol{A}(\boldsymbol{\beta})$, vector $\boldsymbol{b}$, Jacobian $\boldsymbol{J}(\boldsymbol{\beta},\boldsymbol{x}) = \boldsymbol{\nabla}_{\boldsymbol{\beta}}(\boldsymbol{A}(\boldsymbol{\beta})\boldsymbol{x})$, diagonal matrix of positive weights $\boldsymbol{D}$, initial estimate $\hat{\boldsymbol{\beta}}$, minimum sample size k_{min}, maximum sample size k_{max}, number of Monte Carlo experiments M, and tolerance ϵ.

Output: Vector $\boldsymbol{x}$.

begin
  for $i = 1$ to M do
    Get sample size k from discrete uniform distribution $\mathcal{U}\{k_{min}, k_{max}\}$.
    Randomly sample k rows from $\boldsymbol{A}\boldsymbol{x} \approx \boldsymbol{b}$ to form $\widetilde{\boldsymbol{A}}\widetilde{\boldsymbol{x}} \approx \widetilde{\boldsymbol{b}}$.
    for $\widetilde{\boldsymbol{A}}\widetilde{\boldsymbol{x}} \approx \widetilde{\boldsymbol{b}}$ do
      Set $\widetilde{\boldsymbol{\beta}} = \widetilde{\boldsymbol{\beta}_0}$, compute $\widetilde{\boldsymbol{x}}$ from $\underset{\widetilde{\boldsymbol{x}}}{min} \left\| \widetilde{\boldsymbol{b}} - \widetilde{\boldsymbol{A}}\widetilde{\boldsymbol{x}} \right\|_2$ with $\widetilde{\boldsymbol{A}} = \widetilde{\boldsymbol{A}}(\widetilde{\boldsymbol{\beta}})$, compute $\widetilde{\boldsymbol{J}}(\widetilde{\boldsymbol{\beta}}, \widetilde{\boldsymbol{x}})$, and set $\boldsymbol{r} = \widetilde{\boldsymbol{b}} - \widetilde{\boldsymbol{A}}\widetilde{\boldsymbol{x}}$.
      repeat
        Solve $\underset{\Delta\widetilde{\boldsymbol{x}}, \Delta\widetilde{\boldsymbol{\beta}}}{min} \left\| \boldsymbol{M} \begin{bmatrix} \Delta\widetilde{\boldsymbol{\beta}} \\ \Delta\widetilde{\boldsymbol{x}} \end{bmatrix} + \begin{bmatrix} -\boldsymbol{r} \\ \widetilde{\boldsymbol{D}}(\widetilde{\boldsymbol{\beta}} - \widetilde{\boldsymbol{\beta}_0}) \end{bmatrix} \right\|_2$, $\boldsymbol{M} = \begin{bmatrix} \boldsymbol{J}(\widetilde{\boldsymbol{\beta}}, \widetilde{\boldsymbol{x}}) & \widetilde{\boldsymbol{A}}(\boldsymbol{\beta}) \\ \widetilde{\boldsymbol{D}} & \boldsymbol{0} \end{bmatrix}$.
        Set $\widetilde{\boldsymbol{x}} := \widetilde{\boldsymbol{x}} + \Delta\widetilde{\boldsymbol{x}}$, $\widetilde{\boldsymbol{\beta}} := \widetilde{\boldsymbol{\beta}} + \Delta\widetilde{\boldsymbol{\beta}}$.
        Compute $\widetilde{\boldsymbol{J}}(\widetilde{\boldsymbol{\beta}}, \widetilde{\boldsymbol{x}})$, $\widetilde{\boldsymbol{A}}(\widetilde{\boldsymbol{\beta}})$, $\boldsymbol{r} = \widetilde{\boldsymbol{b}} - \widetilde{\boldsymbol{A}}(\widetilde{\boldsymbol{\beta}})\widetilde{\boldsymbol{x}}$.
      until $\|\Delta\widetilde{\boldsymbol{x}}\| < \epsilon$ and $\left\|\Delta\widetilde{\boldsymbol{\beta}}\right\| < \epsilon$
  Compute $\boldsymbol{x} = \frac{1}{M}\sum_{i=1}^{M} \widetilde{\boldsymbol{x}}$.
end

For the formulated problem in Eq.(16 - 18), we have

$$\boldsymbol{\beta} = [q_1 \quad \alpha_1 \quad q_2 \quad \alpha_2 \quad \cdots \quad q_m \quad \alpha_m] \tag{22}$$

$$\boldsymbol{J}(\boldsymbol{\beta}, \boldsymbol{x}) = \begin{bmatrix} J_{1,1} & J_{1,2} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & J_{2,3} & J_{2,4} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & J_{m,2m-1} & J_{m,2m} \end{bmatrix} \tag{23}$$

where

$$\begin{aligned} J_{i,2i-1} = \; & x_2 + \alpha_i x_3 + \alpha_i \tan(\alpha_i) x_4 + \alpha_i^2 \tan(\alpha_i) x_5, \\ & i = 1, \cdots, m \end{aligned} \tag{24}$$

$$\begin{aligned} J_{i,2i} = \; & q_i x_3 + q_i \tan(\alpha_i) x_4 + q_i \alpha_i \left(\tan^2(\alpha_i) + 1\right) x_4 + \\ & q \alpha_i^2 \left(\tan^2(\alpha_i) + 1\right) x_5 + 2\alpha_i q_i \tan(\alpha_i) x_5, \\ & i = 1, \cdots, m \end{aligned} \tag{25}$$

IV. Evaluation

A. Flight Data

The operational flight data used in this study are obtained from QAR of a major airline. QAR is an airborne flight recorder designed to provide quick and easy access to raw flight data. Compared to aircraft's flight data recorder (FDR), QAR is able to sample data at much higher rates and for longer periods of time, but it is not designed to survive an accident. QAR records the values of thousands of aircraft and engine parameters during flight and it is an relatively accurate source of operational flight data.

The dataset consists of 120 real flights of several Boeing 777-300ER aircraft, flying different routes with different time lengths. The flight data are prepossessed by extracting the steady level flight segment based on altitude, flight path angle, acceleration etc., and selecting parameters that are analyzed in the proposed method - dynamic pressure, angle of attack, fuel weight (accumulated by fuel flow rate), and finally resampled at 1Hz.

For the improved SNTLS algorithm, we choose the sample size $k_{min} = k_{max} = 30$, number of Monte Carlo experiments $M = 1000$, tolerance $\epsilon = 1e - 2$, and the diagonal weighting matrix $\boldsymbol{D} = \boldsymbol{I}$.

B. Results

Figure IV-B show the percent error of estimated aircraft weight minus fuel weight W_o compared to same weight computed from QAR data W_o^*.

$$Error = \left| \frac{W_o - W_o^*}{W_o^*} \right| \times 100\% \tag{26}$$

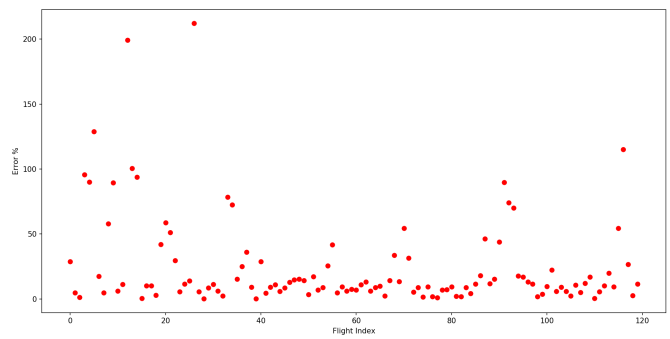

Fig. 1. Percent error of estimated aircraft weight vs. weight computed from QAR data

Figure IV-B shows that the estimated weight W_o for some of the flights are incorrect, while most the estimated weights are with a small fraction of errors compared to the QAR calculated data. This is even more clear in figure IV-B. 24 out of 120 flights' weight estimations are within 5% of error. 87 out of 120 flights' weight estimations are within 20% of error.

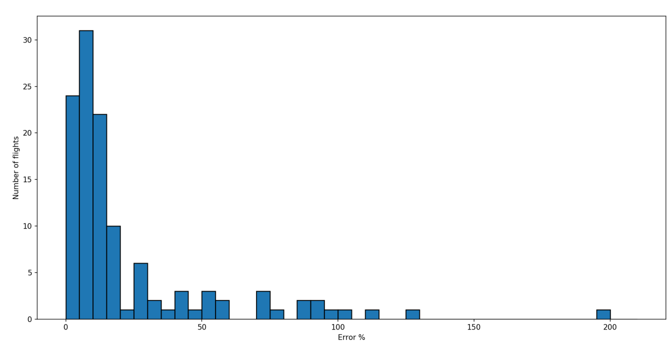

Fig. 2. Histogram of estimation errors

Figure IV-B and figure IV-B shows the angle of attack and dynamic pressure of the flight with best and worst estimation.

These two parameters have more variations for the flight with the best estimation compared to the worst one.

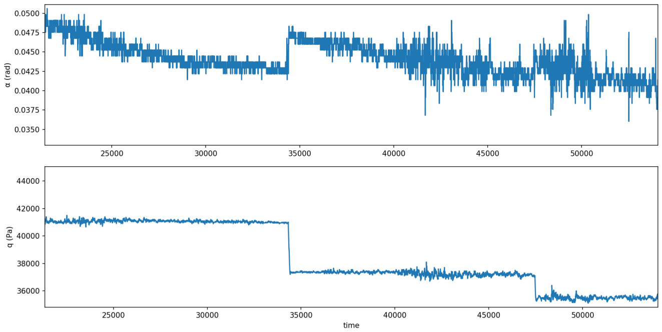

Fig. 3. Parameters of the flight with best estimation

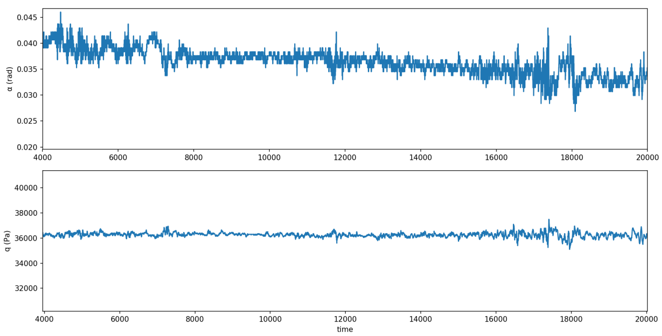

Fig. 4. Parameters of the flight with worst estimation

C. Discussion

There are many problems that can cause large errors in the proposed method. First, we consider the aircraft is in steady level flight state, but there may be small perturbations in acceleration, flight path angle, roll angle, etc. This will make the analyzed flight dynamics inaccurate, result in large errors. Second, SNTLS method assume the errors in parameter vector $\boldsymbol{\beta}$ are zero mean. Although this is generally a good assumption, it might not be true for some of sensor. Notice the sensitivity errors in figure IV-B and IV-B. Moreover, to make the system matrix A to have a good condition number, the flight duration needs to be long enough to consume enough fuel and make the right hand vector $\boldsymbol{b}$'s elements have reasonable difference. And the angle of attack and dynamic pressure need to have wider ranges like in figure IV-B. This way the $\boldsymbol{A}$ and $\boldsymbol{b}$ will have enough information to give a good estimation.

V. Conclusion

In this paper, we focus on how to get an accurate estimation of aircraft mass using QAR data. We reformulate the flight dynamics equations to a set of overdetermined linear equations with uncertainties in both system matrix and the right-hand side vector. The proposed method doesn't depend on thrust, and it doesn't require knowledge of aircraft specific information like geometry, aerodynamic coefficients, etc. The set of equations is solved by an improved structured nonlinear total least squares method using Monte Carlo method. The method is applied to 120 real flights of Boeing 777-300ER aircraft, the result shows a good accuracy for flights with longer duration and more information in measured parameters. The proposed method can be further extended to climb or descent flight phases, which will have much more information compared to the cruise phase. This may also make the equations more complicated or even nonlinear. But if it can be solved properly, this will give much accurate estimations.

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